Antenna Selection in Polarized-MIMO System using Interior Point Method

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***Abstract*— By selecting the best M' out of M antennas, antenna selection significantly lowers the hardware complexity of MIMO system with large antenna elements at both link ends. The computational complexity of the widely used antenna selection method is NP-hard. We lower the complexity by formulating the selection problem as a convex optimization problem. We apply this technique to the polarized-MIMO (P-MIMO) system to additionally gain the benefit of polarization diversity to the system. Our simulation result validates that by employing antenna selection to P-MIMO, we can achieve full conventional uni-polarized MIMO capacity with just a subset of antennas. Further, using convex optimization for antenna selection gives a close agreement to that obtained by brute-force numerical search, while yielding lower computational complexity.**

I. Introduction

Multiple input multiple output (MIMO) communication system has been the furnace of wireless communication system for the past recent years. Naturally, much effort has been invested by the leading scholars to enhance the MIMO system. One particular area for enhancement has been to address the high hardware complexity of MIMO which require $N \times M$ number of RF chains for a system with N number transmitters (Tx) and M number of receivers (Rx). In this context, antenna selection is a promising choice that can significantly mitigate this problem because it chooses a subset of antenna elements which captures a large portion of the full MIMO system capacity. By selecting M' out of M Rx antennas, antenna selection reduces the hardware complexity by lowering the number of RF chains of Rx from M to M'.

Another way to enhance MIMO system is to incorporate polarization diversity. Polarization diversity has demonstrated a promising potential to improve MIMO system in terms of symbol error rate (SER) and channel capacity. In particular, [1] describes a polarized-MIMO system that significantly increases the channel capacity from that of the conventional MIMO system. To further enhance the MIMO system, this paper serves to combine polarization diversity and antenna selection by capturing the benefit they both provide.

The advantage of antenna selection is demonstrated in [2], [3]. However, polarization diversity is not taken into account in the majority of previous research works. Although there are previous reports that consider polarization diversity with antenna selection, they consider fixed antenna polarization, [4], [5]. In contrast, we exploit antenna selection with polarization-agile antenna elements which significantly outperforms the conventional scheme of the conventional MIMO system. Further, most of the antenna selection algorithm has high computational complexity, as the antennas are selected with brute-force search. However, this paper formulates and solves the antenna selection problem as a convex optimization problem which yields lower computational complexity.

II. System Model

Polarized-MIMO (P-MIMO) system with antenna selection is illustrated by Fig. 1, where antenna elements change the antenna polarization angles to any continuous degrees. Our objective is to select M' out of M such antenna elements at Rx. The effective channel matrix of P-MIMO system is described as

$$\mathbf{H}^{\text{eff}} = \begin{bmatrix} \vec{p}_{\text{Rx},1}^{\,T} H_{11} \vec{p}_{\text{Tx},1} & \cdots & \vec{p}_{\text{Rx},1}^{\,T} H_{1N} \vec{p}_{\text{Tx},N} \\ \vdots & \ddots & \vdots \\ \vec{p}_{\text{Rx},M}^{\,T} H_{M1} \vec{p}_{\text{Tx},1} & \cdots & \vec{p}_{\text{Rx},M}^{\,T} H_{MN} \vec{p}_{\text{Tx},N} \end{bmatrix}, \tag{1}$$

where the operation $(\cdot)^T$ is the transpose of a given vector or matrix. Further, H_{ij} is called "*polarization-basis matrix*", which is expressed as

$$H_{ij} = \begin{bmatrix} h_{ij}^{\text{vv}} & h_{ij}^{\text{vh}} \\ h_{ij}^{\text{hv}} & h_{ij}^{\text{hh}} \end{bmatrix}, \tag{2}$$

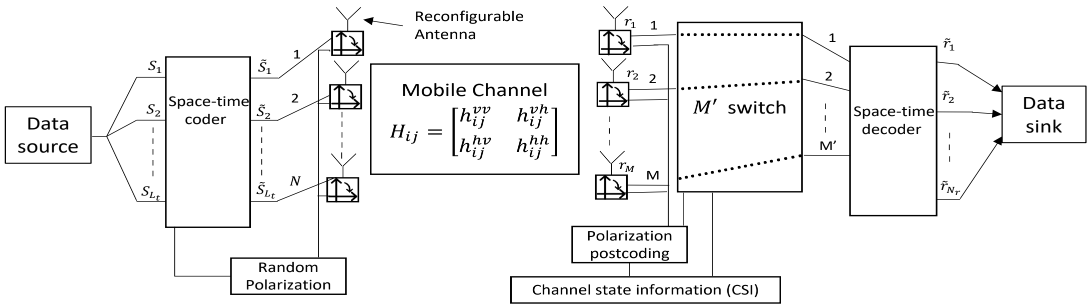

Fig. 1. Antenna selection in P-MIMO system.

where h_{ij}^{xy} with $\mathrm{x} \in \{\mathrm{v}, \mathrm{h}\}$; $\mathrm{y} \in \{\mathrm{v}, \mathrm{h}\}$ is the XY-channel impulse response from the Y-polarization Tx antenna to the X-polarization Rx antenna. Each entry of (2) is modeled as independent identically distributed (i.i.d.) zero-mean, circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. Lastly, $\vec{p}_{\mathrm{Tx},j}$ and $\vec{p}_{\mathrm{Rx},i}$ are, respectively, the Tx-polarization vector at the jth Tx antenna and the Rx-polarization vector at the ith Rx antenna, and they are expressed as

$$\vec{p}_{\mathrm{Tx},j} = \begin{bmatrix} p_{\mathrm{Tx},j}^{\mathrm{v}} \\ p_{\mathrm{Tx},j}^{\mathrm{h}} \end{bmatrix} = \begin{bmatrix} \cos\theta_j \\ \sin\theta_j \end{bmatrix}, \tag{3}$$

$$\vec{p}_{\mathrm{Rx},i} = \begin{bmatrix} p_{\mathrm{Rx},i}^{\mathrm{v}} \\ p_{\mathrm{Rx},i}^{\mathrm{h}} \end{bmatrix} = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}. \tag{4}$$

Here, we call the angles θ_j and θ_i Tx- and Rx-polarization angles, respectively. It is worth mentioning that Tx- and Rx-polarization vectors are unit vectors so that the overall signal power is preserved. Optimal polarization vectors that maximize the sum of squared singular value of (1) is described in detail in [1].

We consider a system where Tx does not know about the channel while Rx does. Then the capacity of P-MIMO system is described by

$$C(\mathbf{H}^{\mathrm{eff}}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{R_{ss}} (\mathbf{H}^{\mathrm{eff}})^H \mathbf{H}^{\mathrm{eff}}), \tag{5}$$

where γ is the signal to noise ratio (SNR), $\mathbf{I}_N$ is the $N \times N$ identity matrix and $\mathbf{R_{ss}}$ is the covariance matrix of Tx signals. Since Tx does not know about the channel, $\mathbf{R_{ss}}$ is chosen as I_N/N and optimal polarization is found only at the Rx while the Tx has its antenna polarization at random angles.

III. Antenna Selection as Convex Optimization Problem

Antenna selection chooses M' out of M receivers. We express (5) with M' selected receivers as

$$C_r(\mathbf{H}_\mathbf{r}^{\mathrm{eff}}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{R}_{ss} (\mathbf{H}_\mathbf{r}^{\mathrm{eff}})^H \mathbf{H}_\mathbf{r}^{\mathrm{eff}}), \tag{6}$$

where the dimension of $\mathbf{H}_\mathbf{r}^{\mathrm{eff}}$ is $M' \times N$. Further, we define (5) as a function of selected antennas by defining Δ_i as

$$\Delta_i = \begin{cases} 1, & \text{if } i^{th} \text{ receive antenna selected} \\ 0, & \text{if otherwise.} \end{cases} \tag{7}$$

Using (7), capacity described by (6) becomes a function of $\mathbf{\Delta}$ as

$$C(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H}^{\mathrm{eff}} (\mathbf{H}^{\mathrm{eff}})^H), \tag{8}$$

where $\mathbf{\Delta}$ is a diagonal matrix consists of Δ_i's. This is derived rigorously in [6].

The objective is to find Δ_i's which maximize (6). We observe that this problem is NP hard because it is solved with brute-force search with $\binom{M}{M'}$ cardinality as described in [7]. We seek to formulate the problem into a simpler problem by applying relaxation on the Δ_i's by allowing $\Delta_i \in [0, 1]$. This problem then becomes a convex optimization problem with lower complexity. The reformulated problem is described as follow

$$\begin{aligned} \text{maximize}: \ & \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H}^{\mathrm{eff}} (\mathbf{H}^{\mathrm{eff}})^H) \\ & \text{subject to} \\ & 0 \le \Delta_i \le 1, \ i = 1, ..., M \\ & \mathrm{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'. \end{aligned} \tag{9}$$

We apply a rounding scheme after the solution is found, where we round the highest M' Δ_i's to 1 and the rest to 0; which indicates the selected antenna. This is solved efficiently using the interior point method [8] which is described in detail in Sec. IV. In Sec. V, we compare the capacity of P-MIMO to conventional MIMO. It is worth to note that conventional MIMO employ $M \times N$ channel matrix $\mathbf{H}$ whose entries are (ZMCSCG); therefore, the capacity of convention MIMO system can be analyzed by replacing $\mathbf{H}^{\mathrm{eff}}$ by $\mathbf{H}$ in (5) and (9).

IV. Interior Point Method

Interior point method solves optimization problems that contain inequality constraints by combining the objective function with a barrier term. Adding such term results in having the optimal unconstrained value in the feasible space, such that violation of inequality constraints could be prevented. The optimization problem in (9) is reformulated using interior point method as follows

$$\text{minimize} : tf_0'(\Delta_i(t)) + \phi(\Delta_i(t))$$
$$\text{subject to}$$
$$\text{trace}(\boldsymbol{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'. \tag{10}$$

where

$$f_0'(\Delta_i(t)) = -\log_2 \det(\mathbf{I}_M + \gamma \boldsymbol{\Delta} \mathbf{H}^{\text{eff}} (\mathbf{H}^{\text{eff}})^H)$$
$$\phi(\Delta_i(t)) = -\sum_{i=1}^{M} \log(\Delta_i(t)(1 - \Delta_i(t))) \tag{11}$$

are the objective and the inequality constraint of (9), respectively.

Newton's method is used to find the optimal value for (10). It takes initial value $\Delta_i(t)$ in the strictly feasible set, and $\Delta_i(t)$ is updated by adding the Newton's step in each iteration. The Newton's step is calculated using the equation as follow

$$\begin{bmatrix} H(\Delta_i(t)) & J(\Delta_i(t))^T \\ J(\Delta_i(t)) & 0 \end{bmatrix} \begin{bmatrix} \Delta_{\text{Newton}} \\ \lambda \end{bmatrix} = \begin{bmatrix} -g(\Delta_i(t)) \\ -h(\Delta_i(t)) \end{bmatrix}, \tag{12}$$

where $f_0(\Delta_i(t))$is the objective function in the optimization problem (10), $h(\Delta_i(t))$ is the equality constraint in (10), $g(\Delta_i(t))$ is the gradient of $f_0(\Delta_i(t))$, $H(\Delta_i(t))$ is the Hessian of $f_0(\Delta_i(t))$, and $J(\Delta_i(t))$ is the Jacobian of $h(\Delta_i(t))$.

Each value $\Delta_i(t)$ is updated by adding Δ_{Newton}, the update parameter μ is used to update t in each iteration. The overall algorithm to update Δ_i using Newton's method is summarized in Algorithm 1.

V. Experiment

In this section, we present the performance of our system with experiment results found via Monte-Carlo simulation. We obtain the average capacity of over 2000 realization of the channel matrix for SNR regime from 0 to 20 dB. The result is illustrated in Fig. 2. The simulation parameters are as follows, $N = 2$, $M = 6$ and $M' = 2$. Fig. 2 conveys that optimally selected antennas (red) yield higher capacity than that of randomly selected antennas (yellow). Moreover, its capacity has very close agreement to that of the capacity of selected antennas found with brute-force search (green). This proves that the convex method performs as well as the brute-force method. By applying (9) to the P-MIMO system (black), the capacity is close to the capacity found with full antennas of conventional MIMO system (blue); therefore, exhibiting that P-MIMO combined with antenna selection further enhanced antenna selection system from that of conventional MIMO system.

Algorithm 1 Update Δ_i

Require: $\Delta_i \ (i = 1, 2, ..., M), \ t > 0,$
  $\mu \ (update\ parameter) > 1, \ \epsilon \ (tolerance) > 0$
Ensure:
  $J(\Delta_i(t)) \leftarrow$ Jacobian $of \ h(t)$
  $h(t) \leftarrow \sum_{i=1}^{M} \Delta_i - M'$
  while $M/t >= \epsilon$ **do**
    $\Delta_{Newton} \leftarrow -J(\Delta_i(t))^{-1} h(t)$
    $\Delta_i \leftarrow \Delta_i + \Delta_{Newton}$
    $t \leftarrow \mu t$
  end while

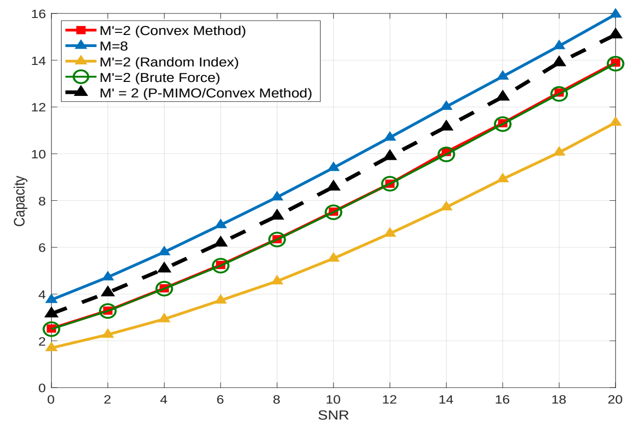

Fig. 2. Capacity v/s SNR, $M = 6$, $N = 2$, $M' = N$

VI. Discussion

The time complexity of each Newton's step is $O(M^3)$, and the total number of Newton's step of the interior point method has an upper bound of $O(M^{0.5})$. To summarize, the time complexity of Algorithm 1 is $O(M^{3.5})$. On the other hand, the exhaustive search method, which selects all possible M' from M antennas, has complexity $O(M^5)$. Therefore, Algorithm 1 provides a solid improvement in time complexity than the exhaustive search.

VII. CONCLUSIONS

This paper finds the best subset of polarization-agile antennas of P-MIMO system which captures the large portion of the full system capacity. Antenna selection problem is formulated into a convex optimization problem which was solved efficiently using interior point method. The result shows that the proposed method yield a capacity that has a very close agreement with the capacity found with brute force search; showing that we have the advantage over the brute force method because our method has lower complexity.

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